ON THE CHANGES IN LENGTH PRODUCED IN IRON WIRES BY MAGNETIZATION

DISSERTATION

PRESENTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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L. T. MORE

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ON THE CHANGES IN LENGTH PRODUCED IN IRON WIRES BY MAGNETIZATION.

By Louis Trenchard More.

THIS investigation was undertaken at the suggestion of Professor Rowland, and has for its object the finding of a relation between the change of length produced in iron wires by magnetization, and the intensity of magnetization existing in the wire. It was hoped thus to obtain results that would be comparable, and to avoid certain errors common to all previous work.

Historical.

That magnetizing an iron rod causes it to alter in length was first discussed by Joule[1] in 1847. His attention was called to the phenomenon by a machinist of Manchester, who imagined that the volume of a mass of iron was increased by magnetizing it. Joule, to test the opinion of the machinist, immersed a mass of iron in a closed vessel full of water in which stood a fine capillary tube. When the iron was strongly magnetized, the height of the column of water in the tube remained unaltered, showing that within the limits of accuracy of his apparatus, for the intensity employed, the volume of the iron was unchanged. Bidwell[2] has also investigated this subject and found, on the contrary, that the volume was altered by magnetization. The volume diminishes at first and attains a minimum; it then increases until with sufficiently intense fields the original size is regained; after reaching this point the volume continues to increase. As a consequence of this relation, if Joule had used an intensity either greater or less, he probably would have noticed a change in the volume. Joule afterwards, by means of a system of levers, found that the length

[1] Joule, Phil. Mag. (3), Vol. XXX., pp. 76, 225.
[2] Bidwell, Proc. Roy. Soc., Vol. LVI., p. 94.

of a rod was increased by the magnetizing force, and gave as a result of his observations the following laws: —

1. When soft iron rods are magnetized, their length is increased and the elongation is approximately proportional to the square of the magnetizing force.

2. Tension applied to the rod diminishes the elongating effect, and — "In the case of a bar one foot long and one-quarter inch in diameter, a tensile force of about 600 pounds caused all the phenomena of changes of length to disappear."

3. "That the elongation is greater, for the same intensity of magnetism, in proportion to the softness of the metal. It is greatest of all in the well annealed iron bars, and least in hardened steel. This circumstance appears to me to favor the hypothesis that the phenomena are produced by the attractions taking place between the magnetized particles of the bar, an hypothesis in perfect accordance with the law which I have pointed out," — that the elongation was proportional to the square of the intensity of magnetization.

The first two laws pointed out by Joule have been often confirmed, but the third seems to rest on a single experiment, and until very recently there have been no published records, that I have seen, bearing on the result found by Joule. Shelford Bidwell[1] while investigating the subject obtained results the converse of Joule's, — that not only hardening, but also annealing, the iron diminished the elongating effect. He mentions one specimen that when annealed contracted in length instead of elongating upon the application of the feeblest magnetizing force.

Barrett[2] in 1870 discovered that nickel when magnetized contracts instead of elongating. Three years later A. M. Mayer[3] published an account of his experiments on this subject. His results in the main verified Joule's observations, with the exception of the action of hard steel. This discrepancy was shown some years later by Bidwell to follow from their different methods of experimenting. Joule applied a current of the same intensity but once, and both on making and on breaking the circuit observed an elongation; while Mayer used specimens already perma-

[1] Bidwell, Proc. Roy. Soc. Vol. LV., p. 228. [2] Barrett, Nature, 1882.
[3] Mayer, Phil. Mag. Vol. XLVI., p. 179.

nently magnetized, and on observing the temporary magnetization, found a contraction on making circuit and an elongation on breaking it.

Mayer also observed hysteretic effects; that is, the elongation due to a magnetizing force was less if the force had been reached by successively increasing values, than it was if the current had been decreased from a maximum, the rod remaining slightly elongated after the magnetizing force had been removed, an effect analogous to the lagging of the induction behind the magnetizing force. Nagaoka[1] has discussed this phenomenon in the article cited, and has obtained complicated curves showing the complete cycle of the hysteretic phenomena for both iron and nickel.

The experiments mentioned were limited to comparatively weak fields; the work of finding the effects due to intense fields has been most thoroughly done by Shelford Bidwell,[2] who has found that rods do not continue to elongate indefinitely with increasing strengths of field, as the other investigators supposed, but that a maximum value is after a time reached. The rod then begins to shorten, and very intense fields produce an absolute contraction which approaches a limiting value asymptotically. He also experimented with rings of iron, and with rods of steel, nickel, manganese steel, cobalt, and bismuth.

Investigations upon this subject have also been made by Berget,[3] Nagaoka,[4] Lochner,[5] Jones,[6] and Bock.[7]

For convenience, I have collected in a summary the results obtained by the different observers.

Soft Iron. — Soft iron elongates when magnetized. The elongation attains a maximum and then diminishes with increasing strength of field until a state is reached when the rod returns to its original length. Further increase of field causes the rod to contract.

[1] Nagaoka, Phil. Mag., Vol. XXXVII., p. 131.

[2] Bidwell, Proc. Roy. Soc., Vol. XXXVIII., p. 265; Vol. XL., pp. 109, 237; Vol. XLVII., p. 469; Vol. LV., p. 228; Trans. Roy. Soc., Vol. CLXXIX. (A), p. 205.

[3] Berget, Comp. Rend., tom. CXV., p. 722.

[4] Nagaoka, Wied. Ann., LIII., pp. 481, 487; 1894.

[5] Lochner, Phil. Mag., Vol. XXXVI., p. 504; 1893.

[6] Jones, Phil. Mag., Vol. XXXIX., p. 254; 1895.

[7] Bock, Wied. Ann.; 1895.

There is no minimum length, the rod approaching asymptotically a limiting value.

With a given strength of field, both hardening and annealing diminish the elongation and increase the contraction shown by the rod before it was subjected to these operations. (Bidwell.)

Tension also diminishes the elongation and increases the contraction of the rod.

For a sufficiently great tension no elongation occurs, the rod contracting upon the application of the smallest magnetizing forces.

For a given length, the effects both of elongation and of contraction are greater for thin than for thick rods. (Bidwell.)

S. J. Lochner[1] comes to the conclusion from his own experiments that the converse is true, — that thick bars show greater elongation than thin ones.

Very little reliance can be put in these last experiments, and the dependence of the change of length upon the ratio of the length to the diameter cannot be inferred from them. Bidwell used three rods 10 cm. long and 2.65, 3.65, and 6.25 mm. in diameter, and assumed them to be of iron of similar composition. It is well known that different specimens of iron, apparently similar in structure, give results that vary 25 per cent and more, so that the small variations in the change of length noted by him cannot safely be said to be due to the differences in their diameters, especially as he made no determination of the permeability. Lochner avoided this error by testing an iron rod, and then after having cut off a portion, testing it again. He, however, took no precaution to have the field uniform. His solenoid was nearly four times as long as the shortest rod used, and the ratio of the diameter to the length of the rod was only 1 to 32. Besides the uncertainties introduced by such a poor arrangement for a uniform field, great errors would be produced by the strong poles created at the ends of the rod.

Object of Experiment.

A careful study of the results obtained for iron by the different observers will show that although the general appearance of their

[1] Lochner, *loc. cit.*

curves is very similar, yet their absolute values vary widely, two specimens often having maximum elongations that differ 20 or 30 per cent. There is, unfortunately, no way of comparing these results, for it has been the custom to use as co-ordinates the change of length and the intensity of the external field. For a given apparatus this intensity depends only on the current used, and not at all on the specimen to be examined. The elongation is dependent upon the intensity of magnetization in the wire, and this is the quantity that varies with the specimen employed. For that reason the relation should be found between these two quantities.

It is, of course, essential to have the rod uniformly magnetized throughout its length; that is, the field should be uniform and there should be no free poles. These conditions may be best obtained in one of two ways, either by having the metal in the shape of a ring, and observing the change in diameter of the ring when magnetized by a solenoid wound upon it, or by using long wires of the metal. In the second case only the middle part of the wire should be observed, and the solenoid used to magnetize it should be considerably longer than the portion of the wire experimented upon. In this investigation the latter method was chosen, as rings are less convenient, and also because it was desirable to observe the effects of tension in the metal.

When a rod of iron is magnetized, the change in length observed is due to several causes, — three at least, — and to obtain a correct idea of the phenomena these causes and their effects should be separated. There is first the *direct action* of the magnetism, and this may possibly be due to the orienting of the magnetized particles of the rod. Secondly, there are indirect actions of the magnetism which tend to change the length of the rod. These indirect actions are the mechanical stresses created in the rod by the magnetism. The first of these mechanical stresses is the tractive force of the magnet and is measured by $\frac{B^2}{8\pi}$. That this force exists, tending always to contract the rod, is seen from the fact that if the magnet is cut in two, the ends are held together by a force $\frac{B^2}{8\pi}$ per square centimeter, showing that this force

must always be present when a rod is magnetized. The agreement between the theoretical value $\frac{B^2}{8\pi}$ and the experimental law for the lifting power has been recently shown by E. Taylor Jones.[1] This effect for high intensities of magnetization is a large one, and becomes one of the most important factors in the observed changes in length. The second of these mechanical stresses is the effect due to the change in Young's modulus when the rod is magnetized. That the elasticity is influenced by magnetization was shown theoretically by J. J. Thomson,[2] and the phenomenon was observed last year in some experiments made by the writer; but no quantitative results could be obtained beyond the fact that the elasticity for soft iron was slightly diminished. A. Bock[3] concludes from his work that the decrease in elasticity must be less than one-half per cent. If the wire is not stretched by weights, this decrease in the elasticity will affect only the contraction due to the $\frac{B^2}{8\pi}$ force. On the other hand, if the wire is loaded with weights, this effect becomes very marked, since a large quantity, the stretch of the wire by the great weight, is altered. This question will be more fully discussed later in the paper.

If these two indirect actions were allowed for, there would remain only the direct action of the magnetism upon the metal under a constant tension. This latter relation would evidently furnish comparable results, and may in the future throw some light upon the action of magnetism on matter.

Apparatus.

Since it was necessary to obtain the modulus of elasticity for the specimen experimented on, in order to make a proper correction for the electro-magnetic stress, and as I wished to observe the effects of mechanical stress, it was convenient to experiment on thin, long wires of the metal placed in a vertical position. To magnify the phenomena a system of levers was used, involving the

[1] Jones, *loc. cit.*

[2] J. J. Thomson, Application of Dynamics to Physics and Chemistry, p. 58.

[3] Bock, *loc. cit.*

tilting of a mirror mounted on three legs very close together, — a method first invented by Professor Rowland. The general plan of using levers and a jacket cylinder was suggested to me by Dr. Ames.

The wire to be tested was suspended from a tall tripod standing on a stone slab that rested on two brick piers. A hole was cut in the slab so that the wire could be passed through it and the free end permitted to almost reach the floor of the room. A hollow brass cylinder 1.6 cm. in diameter and about 68.0 cm. long (Fig. 1) was screwed to the wire at a point *a*, a short distance above the stone slab; a loosely fitting cork plug in its open upper end served to keep the wire in the axis of this cylinder. To the cylinder was screwed the brass arm *deb*. At *e* was screwed a hard steel support for the fulcrum of the lever. The side view of the support was of this shape, |_____|, the two projections supporting the knife edge, and the body of the lever passing between them. To keep the lever horizontal and to register changes of length, another knife edge with its blade upwards was embedded in the lever. This knife edge pressed against the piece *mn*, which was screwed to the wire at *m*. Now if the points *m* and *a* were separated or brought close together, the knife edges would no longer remain in a horizontal line, and the lever would tilt. The end *p* of the lever, and the surface *d* of the arm *bed*, were made planes to support a small brass table having on its upper face a vertical plane mirror, and for legs three short bits of needles, two of which stood on *p* and one on *d*. By this arrangement the change in the length of *ma*, already magnified by the lever, was much more apparent when the tilting of the mirror table was read by means of a telescope and scale. The most important dimensions are: —

Length of wire stretched, *ma*	69.9	cm.
Ratio of lever arms	$\frac{119}{4.65}$	
Distance between needle points . . .	2.3	mm.
Distance to telescope and scale . . .	1660.0	mm.

The multiplying power of the apparatus was, therefore, —

$$\frac{119}{4.65} \times \frac{1660}{2.3} \times 2 = 36941.$$

And as the scale was graduated to millimeters, one scale division represented an actual change of length of 2.7×10^{-5} mm.

The coil to magnetize the wire stood on the stone slab and was long enough to reach just below the arm *ce*, so that the part of the wire *ma* experimented on was in a practically uniform field.

The principal dimensions of this coil were as follows: —

Length of coil	83.7	cm.
External diameter	6.3	cm.
Internal diameter	3.6	cm.
Number of wire	18.	
Number of turns	3045.	
Number of layers	7.	
Number of turns per cm.	36.4	
Resistance of coil	10.53	ohms
Strength of field per ampère	45.75	C.G.S.
Maximum field	260.00	C.G.S.

The core of this coil was made of two co-axal brass cylinders fastened together by the end plates of the coil. The space between these two cylinders was filled with water, which proved to be an excellent way of retarding the heat effects produced by the current.

Method.

I at first intended to measure the elongation and the corresponding induction simultaneously, but found this difficult, and so adopted a more convenient and apparently as accurate a method. This was to first measure the current used to produce a given intensity of magnetism and the consequent changes in length, and afterwards to get the relation between the strength of field and the intensity of magnetization in the iron, the apparatus in the meanwhile remaining untouched. As the laboratory is situated in the city, the work had to be done late at night after traffic had stopped; for, in spite of all the precautions that could be thought of, the shaking of the apparatus could not be prevented. For this reason the induction was not determined immediately after obtaining the elongation curve, but was done the next morning. The

induction was measured by the method of reversals.[1] The induction coil used for this purpose consisted of 200 turns of No. 36 wire wound in one layer on a paper cylinder slightly greater in diameter than the specimen. This little cylinder was slipped over the wire and fixed half-way between *am*. The galvanometer was calibrated by means of a standard coil having a wooden core. The secondary of the standard coil also consisted of 200 turns of wire, which plan saved much computation. The intensity was then calculated by the formula $B = H + 4\pi I$. Knowing the change in length and the intensity of magnetization for any current, of course the relation between these two quantities could be easily platted. The current was supplied by a battery of storage cells, and the resistance was regulated by a slide resistance of copper sulphate.

Results.

My first results were obtained from a specimen of moderately soft commercial iron wire, one millimeter in diameter. This wire was free from stress, except that due to the jacket cylinder screwed to it, which weighed 350 grams. The elongations given (Table I.) are those due to temporary magnetism, and each value is the mean of two or three readings which did not vary by more than one per cent. The current was applied suddenly, and as soon as a reading was taken the contact was broken. This operation was then repeated, the current being increased each time, and readings taken until the maximum current was reached. By this means temperature effects, which are relatively slow to act, were avoided.

Figure 3, *C* shows the relation between the elongation per unit length, $\frac{\delta l}{l} \times 10^7$, and the intensity of the field, *H*. This curve is seen to closely resemble in form those given by Shelford Bidwell. When the relation is expressed between change of length and intensity of magnetization, the curve takes the very different form given by Fig. 3, *B*. The wire slowly increases in length until an intensity of about 800 is reached. From that point, until a maximum length is attained, at an intensity of 1200, the elongation is

[1] Ewing, Lond. Electrician, April, 1894.

more marked. After reaching this maximum it rapidly contracts, the last portion of the curve being approximately a nearly vertical straight line. The figure also shows that the point of maximum length does not correspond to the point of greatest permeability of iron, as is seen by comparing this curve with the curve for permeability plotted on the same figure.

It is now necessary to make corrections for the contraction due to the $\frac{B^2}{8\pi}$ force. This contraction is obtained from the formula

$$\frac{\delta l}{l} \times 10^7 = \frac{B^2/8\pi}{M},$$

where M is the modulus of elasticity. My apparatus was well fitted for measuring the latter quantity. A weight of about five kilograms was hung on the wire, and the elongations due to the additions of weights of 20, 50, and 100 grams were read. From these Young's modulus was calculated in the usual manner. It was found to be 2.2×10^{12} for the specimen used. The contractions due to the $\frac{B^2}{8\pi}$ force were then calculated, and are given in column 6 of Table I.

The last column of this table was found by taking the difference between the observed elongation and the $\frac{B^2}{8\pi}$ contraction. The relation between this corrected elongation, due to what has been called the direct action of the magnetism, and the intensity of magnetization, is graphically shown by Fig. 3, *A*. The effect of this correction is to make the elongation much greater for a given intensity. The maximum value of the elongation is more than twice as great as the observed maximum. And the greatest intensity employed, 1300 C.G.S. units, produces an elongation and not a contraction as observed.

No correction for the change in elasticity was made. For the present case it would be insignificant. See discussion of errors at end of this paper.

TABLE I.

H	μ	I	B	$\frac{\delta l}{l}$ (obs.)	$\frac{\delta l}{l}\left(\frac{B^2}{8\pi}\right)$	$\frac{\delta l}{l}$ (corr.)
4.6	384.8	141	1770	+0.10	−0.7	+0.80
7.3	507.0	294	3700	—	—	—
10.5	752.4	623	7900	1.16	11.3	12.46
11.8	733.0	700	8800	—	—	—
12.8	726.7	739	9300	2.71	15.65	18.36
15.5	695.0	856	10770	5.71	20.98	26.69
19.7	627.0	981	12350	14.71	27.60	42.31
24.2	541.4	1041	13100	18.78	31.05	49.83
28.8	482.7	1104	13900	22.92	34.95	57.87
37.6	389.1	1163	14630	25.55	38.53	64.13
53.0	286.3	1203	15170	24.78	41.69	66.47
60.4	253.7	1214	15320	23.23	42.47	65.70
67.7	227.7	1222	15410	21.30	42.96	64.26
80.0	193.8	1228	15500	18.74	43.46	62.20
93.3	167.2	1235	15600	15.49	44.02	59.51
103.8	150.8	1238	15650	12.81	44.30	57.11
111.6	140.7	1241	15700	11.03	44.59	55.62
121.2	130.0	1244	15750	9.20	44.88	54.08
145.4	109.6	1257	15930	5.42	45.90	51.32
159.1	100.6	1261	16010	+1.94	46.37	48.31
186.5	86.9	1274	16200	−2.94	47.47	44.53
210.3	77.8	1281	16360	6.97	48.42	41.45
246.9	67.1	1299	16570	12.97	49.67	36.70
260.6	63.9	1304	16650	16.65	50.15	33.30

The effect of hardening the wire was next considered. A piece of the same quality of iron was heated to a bright red by passing a current through it, and then suddenly cooled. After the operation the wire was much harder, and only slightly burnt on the surface. The observed changes in length, those due to $\frac{B^2}{8\pi}$ and the corrected values are given in Table II. Hardening the iron makes the elongation smaller, and for the present specimen the least intensity of magnetization caused the iron to contract (Fig. 4, B). The corrected relation between the change of length and the intensity is shown by B'. On this plate is also plotted curve A, the same relation, taken from Fig. 3, for the wire when not hardened; and curve A' shows the corrected relations for the same.

Although the observed change for the hardened wire was a contraction, the contraction due to the $\frac{B^2}{8\pi}$ force was sufficient to bring the values above the zero axis. So, again, the direct action of the magnetization is to elongate the wire, and the corrected curve is very similar to the curve for the iron in its original state. The absolute values are, however, much diminished, and the maximum occurs, for this specimen, with a less intensity. The correction for the change of elasticity can be neglected as in the first case.

TABLE II.

H	I	B	$\frac{\delta l}{l}$ (obs.)	$\frac{\delta l}{l}\left(\frac{B^2}{8\pi}\right)$	$\frac{\delta l}{l}$ (corr.)
5.0	8	100	−0.00	−0.00	+0.00
8.7	53	670	0.05	0.08	0.03
12.8	272	3420	0.10	2.12	2.02
18.3	709	8920	0.20	14.41	14.21
27.4	923	11620	0.50	24.44	23.94
36.6	947	11940	0.70	25.77	25.07
45.7	1012	12770	1.85	29.48	27.63
57.2	1044	13180	3.10	31.42	28.32
68.5	1060	13390	4.10	32.42	28.32
96.5	1074	13600	7.40	33.43	26.03
118.8	1064	13490	5.44	32.93	27.53
138.5	1100	13560(?)	15.00	34.24	20.24
180.1	1126	14320	24.20	37.11	12.91

To find the effect of strain the wire was loaded with a weight of 750 kg. per square centimeter, and the relation between the elongation and the intensity of magnetization obtained as in the previous cases. The wire was then loaded with a weight of 1750 kg. per square centimeter, and the same relations found. The values are given in Tables III. and IV. The effects of strains are also shown graphically by Fig. 5, for the observed and corrected values. By consulting the figure it is seen that straining the wire produces the same change that hardening it does. The values are reduced, and the maximum points occur with less intense magnetization. The curves in this figure, as well as in the two former figures, marked no load, have an inaccuracy due to the

strain caused by the weight of the jacket cylinder, and should have ordinates a trifle greater.

Although the change in elasticity has hitherto produced little effect, as soon as the wire is much strained this correction becomes very important, and will modify the curves materially. This alteration in the curves it has not been possible to make, as the relation between the change in elasticity and the intensity of magnetization is unknown. However, from Bock's results some approximations may be arrived at; and these will be considered when the errors likely to occur in this investigation are discussed.

TABLE III.

H	I	B	$\frac{\delta l}{l}$ (obs.)	$\frac{\delta l}{l}\left(\frac{B^2}{8\pi}\right)$	$\frac{\delta l}{l}$ (corr.)
6.9	636	8000	+0.12	−11.58	+11.70
11.4	918	11550	0.62	24.14	24.76
18.5	1080	13600	2.63	33.46	36.09
24.2	1109	13960	3.73	35.27	39.00
30.2	1139	14340	3.87	37.19	41.06
34 7	1150	14480	3.48	37.94	41.42
43.9	1167	14700	2.42	39.09	41.51
55.8	1184	14930	+1.20	40.33	41.53
73.2	1204	15200	−0.39	41.79	41.40
81.8	1212	15300	1.16	42.35	41.19
100.6	1223	15470	4.41	43.30	38.89
122.5	1235	15640	9.10	44.24	35.14
148.1	1247	15820	14.13	45.27	31.14
157.7	1253	15910	15.80	45.80	30.00
175.1	1265	16070	18.58	46.71	28.13
192.1	1276	16230	25.05	47.65	22.60
228.6	1287	16400	29.03	48.65	19.62
251.4	1293	16500	32.13	49.26	17.13

Possible Errors.

It has been shown that the change in elasticity occurring when an iron wire is magnetized modifies the relation between the intensity of magnetization and the elongation; and it has been assumed that, except for heavily loaded wires, this effect will be

very small. Bock[1] found as a result of his experiments that magnetizing soft iron made it more incompressible, and that the change in elasticity was less than one-half per cent. He could not find any relation between the intensity of magnetization and change

TABLE IV.

H	I	B	$\frac{\delta l}{l}$ (obs.)	$\frac{\delta l}{l}\left(\frac{B^2}{8\pi}\right)$	$\frac{\delta l}{l}$ (corr.)
5.5	581	7300	−0.19	−9.64	+9.54
9.1	922	11600	0.58	24.35	23.77
13.7	1011	12720	1.26	29.27	28.01
21.5	1070	13470	2.90	32.83	29.93
27.4	1100	13860	4.06	34.76	30.70
36.6	1134	14280	5.90	36.89	30.99
45.7	1155	14560	7.36	38.36	31.00
54.9	1167	14710	9.68	39.14	29.46
73.2	1178	14880	13.84	40.05	26.21
86.9	1186	14990	16.64	40.65	24.01
96.0	1190	15050	19.74	40.97	21.23
111.1	1198	15160	23.63	41.58	17.95
128.0	1207	15290	27.87	42.29	14.42
139.4	1212	15370	32.51	42.74	10.23
150.9	1217	15440	33.97	43.12	9.15

in elasticity, so that it is impossible to give more than a probable maximum to the correction it is necessary to make in the curves given in this investigation.

If we take $B = 20{,}000$,

$$\frac{B^2}{8\pi} = \frac{1}{2\pi} \times 10^8 \text{ dynes per square centimeter,}$$

and

$$\frac{\delta l}{l} \times 10^7 = -\frac{\frac{1}{2\pi} \times 10^8}{M} = -73.$$

A change of one-half per cent in $\frac{\delta l}{l}$ would give a correction of $0.005 \times 73 = .4$ of a unit. This correction may therefore be neglected, as errors greater than this would occur in measuring the induction.

[1] Bock, *loc. cit.*

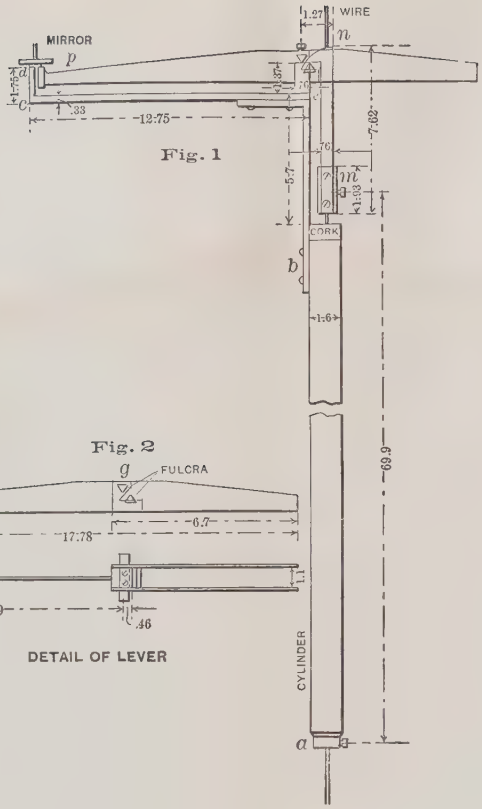
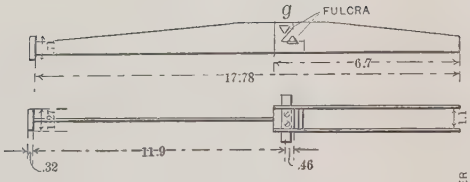


Fig. 1



DETAIL OF LEVER

Fig. 2

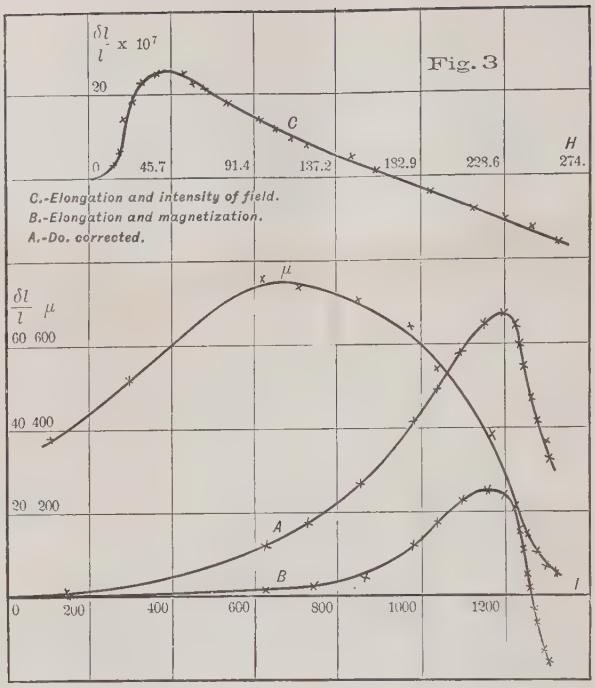


Fig. 3

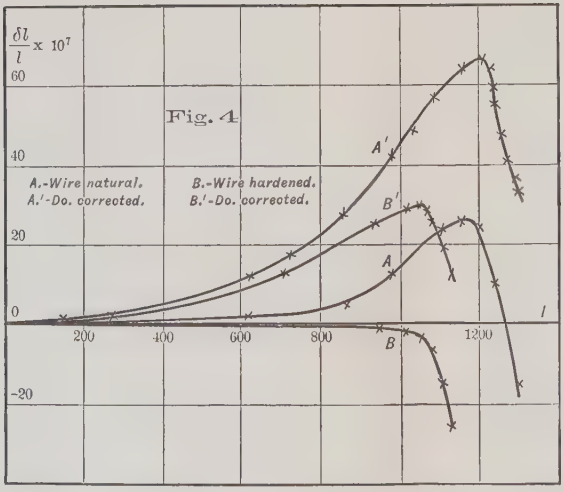


Fig. 4

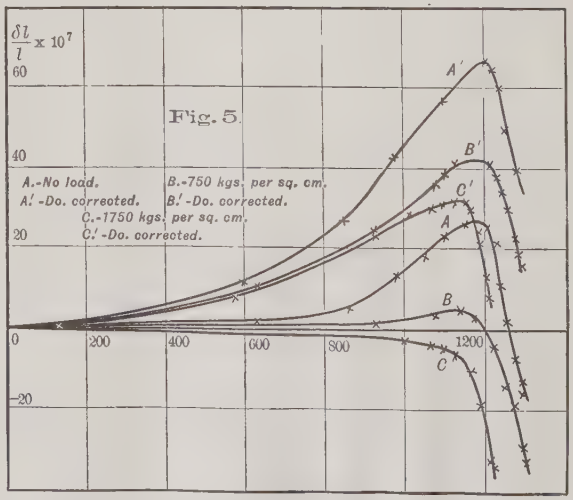


Fig. 5

MORE: CHANGES IN LENGTH DUE TO MAGNETIZATION.

It was also assumed that when the wire was not stretched, except by the jacket, the correction might be neglected.

The weight of this jacket was 350 grams, or 42000 dynes per square centimeter; then

$$\frac{\delta l}{l} \times 10^7 = \frac{42000}{M} \times 10^7 = 200.$$

A change of one-half per cent in 200 is 1.0. Consequently the curves marked no load are not significantly changed by making this correction.

But when we consider the case of the wire loaded with 750 kg. per square centimeter, we get, by a similar calculation, a correction of 15 units; and for the wire under the greatest tension, 1750 kg. per square centimeter, a correction of 40 units. Since the magnetism makes the metal more incompressible, this correction enters as a contraction, and must be added to the ordinates of the curves plotted on Fig. 5. It is impossible to say how these curves would be affected by the correction, but probably the correction increases with the intensity of magnetization.

Changes in the length of the wire due to variations in temperature are another possible source of error. These changes take place more slowly than the elongation produced by the magnetization, so that the two effects may be separated. To retard them still more, the core of the solenoid was made of two co-axal cylinders, and the space between them filled with water. With this arrangement the current could be applied, the reading taken, and the current shut off, before the gradual change of the zero point due to the heating of the wire by the current could be observed. Changes in the temperature of the room when they did occur were too slow to noticeably affect the results.

Summary.

The following is a summary of the results obtained by this investigation: —

1. A relation has been obtained between the elongation due to magnetization and the intensity of magnetization for soft iron. When the elongation has been corrected for the contraction caused

by the $\frac{B^2}{8\pi}$ force, the relation may be expressed thus: A rod of iron elongates slowly until an intensity of about 800 is reached. After that point the rod elongates more rapidly and attains a maximum value of about 60×10^{-7} part of its length at an intensity of 1200. With greater intensities the elongation diminishes, the curve being approximately a nearly vertical straight line.

2. Hardening the wire diminishes the corrected elongation, and the wire attains its maximum length in a less intense field.

3. It has been shown that when the wire is stretched, the change in elasticity due to the magnetization produces an important effect. Neglecting this correction, the effects of stretching are similar to those caused by hardening the wire.

Before concluding I would acknowledge my indebtedness to Professor Rowland, not only for suggesting the investigation to me, but also for his assistance and kind consideration afterwards.

I also owe many thanks to Dr. Ames and to Dr. Duncan for advice and help.